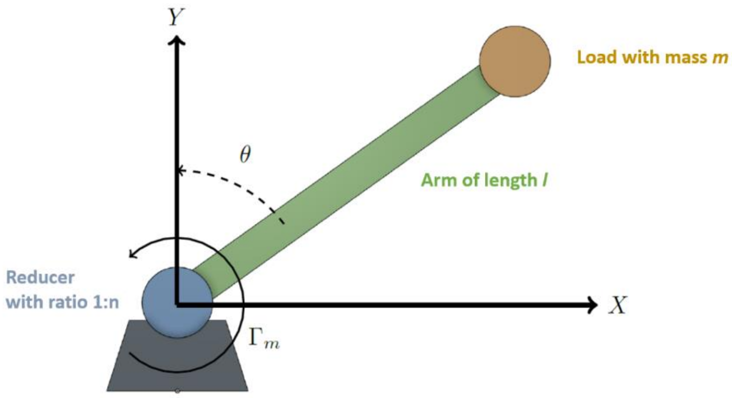
Exercise set 10 – Control – Solutions

Exercise 1

The most common rotational axis model in robotics corresponds to a rotary motor with a reducer, an arm, and a load at the end. If we do not consider the couplings, all the robot arms can be represented by the model shown in the figure below.



Consider:

- J_m the moment of inertia of the motor.

- n the reduction ratio.

- a negligible inertia of the reducer.

- J_a the moment of inertia of the arm about an axis parallel to z and passing through the center of gravity of the arm.

- m_a the mass of the arm.

- l the length of the arm.

- m the load at the end of the arm.

- k_{vis} the coefficient of viscosity referred to the load side.

1. Write the inverse dynamic model of this axis referred to the motor side.
2. Write the simplest control law that can be used to appropriately control this axis in position without static error. Consider that the mechanical viscosity is low.
3. Give the expression of the control law which includes an a priori torque for the control of this axis.
4. Give the expression of a control law of this axis which realizes an exact compensation of the nonlinearities?

Exercise 1 – Solution

1. The arm is governed by the following equation (referred to the motor side):

$$\sum M = J_{tot\_m}\ddot{\theta}_m = \Gamma_m - \frac{1}{n}\left\{(k_{vis}\dot{\theta}_L) + m_a g\frac{l}{2}sin(\theta_L) + mglsin(\theta_L)\right\}$$

J_{tot_m} is the total moment of inertia referred to the motor side:

$$J_{tot\_m} = J_m + \frac{1}{n^2}\left(\left\{J_a + m_a\left(\frac{l}{2}\right)^2\right\} + ml^2\right)$$

The moment of inertia of the arm about the rotational axis of the motor is found by the Huygens-Steiner theorem (parallel axis theorem).

$\dot{\theta}_L = \frac{\dot{\theta}_m}{n}$ and $\theta_L = \frac{\theta_m}{n}$. We therefore have the following equation:

$$J_{tot\_m}\ddot{\theta}_m = \Gamma_m - \frac{1}{n^2}(k_{vis}\dot{\theta}_m) - \frac{1}{n}\left(\frac{m_a}{2} + m\right)glsin\left(\frac{\theta_m}{n}\right)$$

And finally, the IDM of this axis referred to the motor is:

$$\boldsymbol{\Gamma_m = J_{tot\_m}\ddot{\theta}_m + \frac{1}{n^2}(k_{vis}\dot{\theta}_m) + \frac{1}{n}\left(\frac{m_a}{2} + m\right)glsin\left(\frac{\theta_m}{n}\right)}$$

2. The simplest appropriate control law for this axis is a PID:
   - The proportional component is trivially essential.
   - The derivative component is necessary to stabilize the axis (it damps the system).
   - The integral component is necessary to compensate for the gravity torque in static regime. The static regime corresponds to zero speed. In our case, the torque in static regime that must be compensated is:

$$\frac{1}{n}\left(\frac{m_a}{2} + m\right)glsin\left(\frac{\theta_m}{n}\right)$$

   The PID control law is:

$$\Gamma_{reg} = \Gamma_{m\_PID} = K_p\left\{e(t) + T_d\frac{de(t)}{dt} + \frac{1}{T_i}\int_0^t e(\tau)d\tau\right\}$$

   with $e(t)$ the error between the reference and the measurement.

3. The a priori torque Γ_{m_ap} is the motor torque for the position, speed and acceleration setpoint values:

$$\Gamma_{m\_ap} = J_{tot\_m}\ddot{\theta}_{m,d} + \frac{1}{n^2}(k_{vis}\dot{\theta}_{m,d}) + \frac{1}{n}\left(\frac{m_a}{2} + m\right)glsin\left(\frac{\theta_{m,d}}{n}\right)$$

The control law of the PID with an a priori torque is given by the following expression:

$$\Gamma_{reg} = \Gamma_{m\_PID} + \Gamma_{m\_ap}$$
$$= K_p\left\{e(t) + T_d\,\frac{de(t)}{dt} + \frac{1}{T_i}\int_0^t e(\tau)d\tau\right\} + J_{tot\_m}\ddot{\theta}_{m,d} + \frac{1}{n^2}\left(k_{vis}\dot{\theta}_{m,d}\right) + \frac{1}{n}\left(\frac{m_a}{2}+m\right)gl\sin\left(\frac{\theta_{m,d}}{n}\right)$$

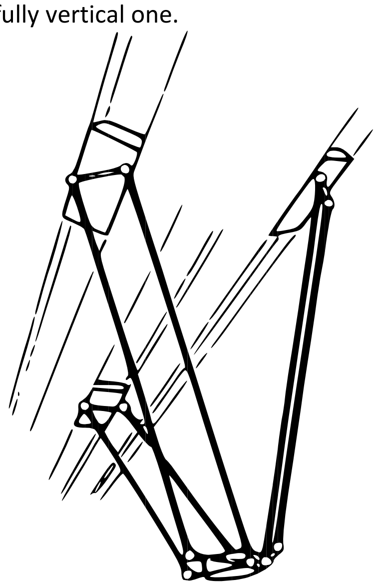
4. The following regulator performs an exact compensation of the non-linearities:

$$\Gamma_{reg} = \Gamma_{m\_PID} + \frac{1}{n}\left(\frac{m_a}{2}+m\right)gl\sin\left(\frac{\theta_{m,d}}{n}\right)$$
$$= K_p\left\{e(t) + T_d\,\frac{de(t)}{dt} + \frac{1}{T_i}\int_0^t e(\tau)d\tau\right\} + \frac{1}{n}\left(\frac{m_a}{2}+m\right)gl\sin\left(\frac{\theta_{m,d}}{n}\right)$$

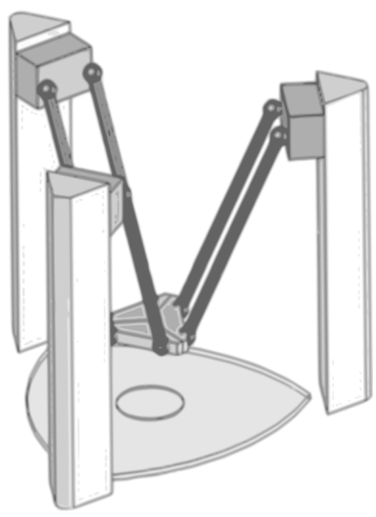
In fact, any regulator which keeps the non-linear terms of the a priori torque, in addition to the PID control law, provides an exact compensation of the non-linearities; including the regulator of question 3.

Exercise 2

There are several variations of the Delta Linear robot. One of the variants is the fully horizontal one and another one is the fully vertical one.



Horizontal linear Delta



Vertical linear Delta

We want to control the linear axes of each of these machines. The following assumptions are made:

- m_h is the equivalent mass referred to each linear axis of the horizontal variant. It is assumed to be constant whatever the position of the robot. All moments of inertia are assumed to be zero.
- m_v is the equivalent mass referred to each linear axis of the vertical variant. It is assumed to be constant whatever the position of the robot. All moments of inertia are assumed to be zero.
- For the two variants, the dry friction is supposed to be zero. The viscous coefficient of friction is k_{vis} (on the motor side) for each linear axis.

1. Give the number of degrees of freedom of:
   (a) the horizontal variant.

   (b) the vertical variant.

2. Regarding the Jacobian matrix, which of the following expressions is correct?
   (a) The Jacobian matrix of the horizontal variant does not depend on the position of the robot.
   (b) The Jacobian matrix of the vertical variant does not depend on the position of the robot.
   (c) The Jacobian matrix of the two variants depends on the position of the robot.
   (d) The Jacobian matrix of the vertical variant corresponds to the identity matrix.

Hint: As a reminder, you can take a look at the document "LinearDeltaGeometricModelling.pdf" which is on Moodle.

3. Concerning the dynamic model, assuming that the masses m_h and m_v are constant, m_h for the horizontal variant, respectively m_v for the vertical variant, which of the following expressions is correct?
   (a) The dynamic model of each of these two robots is decoupled.
   (b) The dynamic model of each of the robots depends on the terminal position of the robot.
   (c) The dynamic models of the two robots are identical.
   (d) Only the dynamic model of the horizontal variant is decoupled.

4. The motors are controlled in torque/force.
   (a) Explain which is the minimum controller that would work for a sufficiently rigid position control of one of the axes of each variant. Consider that there is very little mechanical viscosity. Also neglect gravity for the horizontal variant.
      i. horizontal: P, PI, PD, PID?
      ii. vertical: P, PI, PD, PID?
   (b) With respect to the controller of question (a) i. :
      i. give the expression of the controller.
      ii. use a control diagram to describe the closing of the control loop.
      iii. explain the used variables.
   (c) Give the expression of the IDM of one of the axes of each variant:
      i. horizontal.
      ii. vertical.
   (d) Give the expression of the a priori generalized torques for:
      i. one of the horizontal axes.
      ii. one of the vertical axes.
   (e) Taking into account the controller of question (a) i. :
      i. draw the diagram of the controller with an a priori generalized torque.
      ii. give the total expression of the generalized control torque
   (f) In the case of using an a priori with one of the axes of the vertical variant, explain which is the minimum necessary controller: P, PI, PD or PID.

Exercise 2 – Solution

1. (a) The horizontal variant has 3 DOF.
   (b) The vertical variant has 3 DOF.

2. In both cases the position of the tool is a quadratic function of the joint positions, because each joint at the joint level generates a spherical cap at the tool level:
   (a) FALSE : The Jacobian matrix of the horizontal variant does not depend on the position of the robot.
   (b) FALSE : The Jacobian matrix of the vertical variant does not depend on the position of the robot.
   (c) TRUE: The Jacobian matrix of the two variants depends on the position of the robot.
   (d) FALSE : The Jacobian matrix of the vertical variant corresponds to the identity matrix.

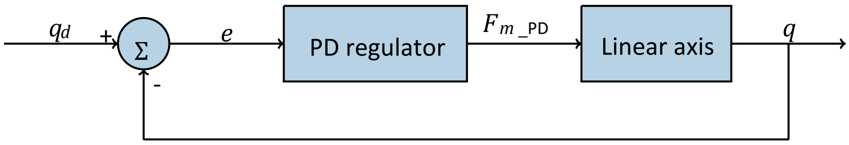
3. The moving masses are assumed to be constant and independent of the position of the robot; the interest of this approximation is to build dynamic models of decoupled axes:
   (a) TRUE: The dynamic model of each of these two robots is decoupled.
   (b) FALSE : The dynamic model of each of the robots depends on the terminal position of the robot.
   (c) FALSE : The dynamic models of the two robots are identical.
   (d) FALSE : Only the dynamic model of the horizontal variant is decoupled.

4. (a) i. The minimum axis controller of the horizontal variant would be a PD controller because the intrinsic viscosity of the axis is not enough to stabilize the closed loop behavior while ensuring good control rigidity. In the case of the horizontal variant, we neglect the effect of gravity. Therefore, there is no need for an integral component.
   Note that we are talking about the minimal controller.
   ii. The minimum axis controller of the vertical variant would be a PID controller because compared to the horizontal variant, we consider the effect of gravity. There is therefore a need for an integral component.

   (b) i. Control law:

$$\mathrm{F}_{m\_PD} = K_p \left\{ (q_d - q) + T_d \frac{d}{dt}(q_d - q) \right\}$$

   ii. Control diagram:



   (Learn to draw it by hand ;-))

   iii. q_d is the desired value of the position of one of the horizontal axes.
   q is the measured value of the position of one of the horizontal axes.

   (c) i. $F_{m,h}$, the driving force of one of the horizontal axes, is equal to:

$$F_{m,h} = m_h \ddot{q} + k_{vis} \dot{q}$$

   ii. $F_{m,v}$, the driving force of one of the vertical axes, is equal to:

$$F_{m,v} = m_v\ddot{q} + k_{vis}\dot{q} + m_v g$$

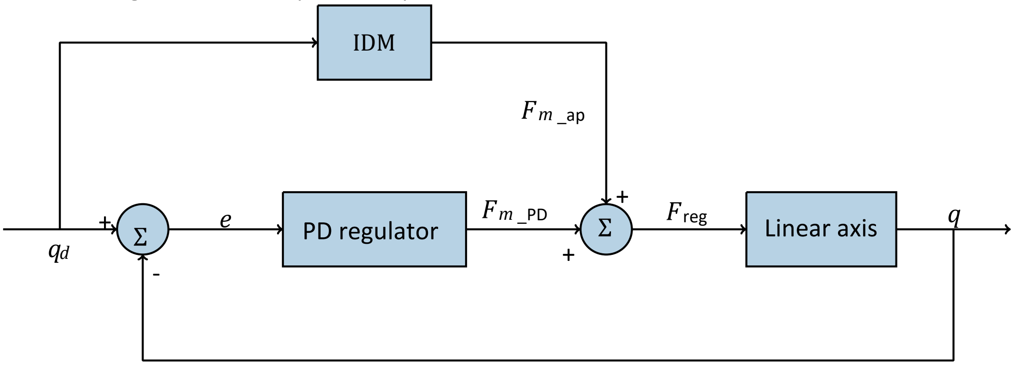
(d) i. $F_{m,\,h\_\mathrm{ap}}$, the a priori driving force of one of the horizontal axes, is:

$$F_{m,h\_ap} = m_h\ddot{q}_d + k_{vis}\dot{q}_d$$

ii. $F_{m,\,v\_\mathrm{ap}}$, the a priori driving force of one of the vertical axes, is:

$$F_{m,v\_ap} = m_v\ddot{q}_d + k_{vis}\dot{q}_d + m_v g$$

(e) i. The control diagram with an a priori torque is as follows:



ii. The control law is:

$$F_{reg} = \mathrm{F}_{m\_PD} + \mathrm{F}_{m,h\_ap} = K_p\left\{(q_d - q) + T_d\,\frac{d}{dt}(q_d - q)\right\} + m_h\ddot{q}_d + k_{vis}\dot{q}_d$$

(f) The minimum controller is a PD because the gravity would be compensated by the a priori.